

কোন: মুক্তকৃত চিহ্ন

সেইভাবে বসালে, $g(x) = \tan^{-1} x$

$$\therefore g(5) = \tan^{-1} 5 \quad \text{কিন্তু } g\left(\frac{1}{5}\right) = \tan^{-1} \frac{1}{5}$$

$$\begin{aligned} \therefore \text{L.H.S} &= \sec^2(g(5)) + \operatorname{cosec}^2(g\left(\frac{1}{5}\right)) \\ &= \sec^2(\tan^{-1} 5) + \operatorname{cosec}^2\left(\tan^{-1} \frac{1}{5}\right) \\ &= 1 + \tan^2(\tan^{-1} 5) + 1 + \cot^2\left(\tan^{-1} \frac{1}{5}\right) \\ &= 1 + (5)^2 + 1 + \cot^2(\cot^{-1} 5) \\ &= 1 + 25 + 1 + (5)^2 \\ &= 31 \end{aligned}$$

$$= \text{R.H.S}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

অতএব, $\sec^2(g(5)) + \operatorname{cosec}^2(g\left(\frac{1}{5}\right)) = 31$ (সেইখানে বসানো)

আন: মুক্তকৃত চিহ্ন

সেইভাবে বসালে,

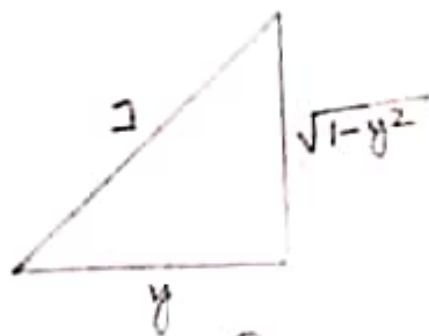
$$f(x) = \sin x \quad \text{কিন্তু } g(x) = \tan^{-1} x$$

$$\therefore f^{-1}(x) = \sin^{-1} x$$

$$\text{সর্বসঙ্গে, } f^{-1}(x) = \cos^{-1} y = \frac{\pi}{6}$$

$$\text{বা } \sin^{-1} x = \cos^{-1} y = \frac{\pi}{6}$$

$$\text{বা, } \sin^{-1} x = \sin^{-1} \sqrt{1-y^2} = \frac{\pi}{6}$$



কোন: চিহ্ন

$$\text{যা, } \sin^{-1} [x\sqrt{1-(\sqrt{1-y^2})^2} - \sqrt{1-x^2} \cdot \sqrt{1-y^2}] = \frac{\pi}{6}$$

$$\text{যা, } x\sqrt{1-y^2} - \sqrt{1-x^2} \cdot \sqrt{1-y^2} = \sin \frac{\pi}{6}$$

$$\text{যা, } xy - \sqrt{(1-x^2)(1-y^2)} = \frac{1}{2}$$

$$\text{যা, } xy - \frac{1}{2} = \sqrt{(1-x^2)(1-y^2)}$$

$$\text{যা, } x^2y^2 - 2 \cdot xy \cdot \frac{1}{2} + \frac{1}{4} = 1 - x^2 - y^2 + x^2y^2 \quad [\text{বর্গ করে}]$$

$$\text{যা, } x^2 + y^2 - xy = 1 - \frac{1}{4}$$

$$\text{যা, } x^2 + y^2 - xy = \frac{3}{4}$$

$$\therefore 4(x^2 + y^2 - xy) = 3 \quad (\text{প্রমাণিত})$$

প্রশ্ন: প্রমাণ করুন

উদাহরণ আছে,

$$f(x) = \sin x$$

$$\therefore f(\pi \cos \theta) = \sin(\pi \cos \theta)$$

$$\text{যে } f\left(\frac{\pi}{2} \pm \pi \sin \theta\right) = \sin\left(\frac{\pi}{2} \pm \pi \sin \theta\right)$$

কাজে,

$$f(\pi \cos \theta) = f\left(\frac{\pi}{2} \pm \pi \sin \theta\right)$$

$$\text{যা, } \sin(\pi \cos \theta) = \sin\left(\frac{\pi}{2} \pm \pi \sin \theta\right)$$

$$\text{যা, } \pi \cos \theta = \frac{\pi}{2} \pm \pi \sin \theta$$

$$\text{যা, } \cos \theta = \frac{1}{2} \pm \sin \theta \quad [\text{উভয় পক্ষকে } \pi \text{ দ্বারা ভাগ করে}]$$

$$\text{যা, } \cos \theta \pm \sin \theta = \frac{1}{2}$$

$$\text{যা, } \cos \theta \frac{1}{\sqrt{2}} \pm \sin \theta \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} \quad [\text{উভয় পক্ষকে } \frac{1}{\sqrt{2}} \text{ দ্বারা গুণ করে}]$$

$$\text{या } \cos \theta \cos \frac{\pi}{4} \pm \sin \theta \sin \frac{\pi}{4} = \frac{1}{2\sqrt{2}}$$

$$\text{या, } \cos(\theta \pm \frac{\pi}{4}) = \frac{1}{2\sqrt{2}}$$

$$\text{या, } \theta \pm \frac{\pi}{4} = \cos^{-1}\left(\frac{1}{2\sqrt{2}}\right)$$

$$\text{या, } \theta \pm \frac{\pi}{4} = \sec^{-1}(2\sqrt{2})$$

$$\therefore 4\theta \pm \pi = 4 \sec^{-1}(2\sqrt{2}) \text{ (प्रधाना द्वारा)}$$

घनः प्रकृतयः पितवः

दृष्ट्या आत्त,

$$f(x) = \sin x$$

$$\therefore f(2x) = \sin 2x$$

$$\text{जिः } f\left(\frac{\pi}{2} - 2x\right) = \sin\left(\frac{\pi}{2} - 2x\right) = \cos 2x$$

$$\text{विधान, } \frac{1}{f(2x)} - \frac{\sqrt{3}}{f\left(\frac{\pi}{2} - 2x\right)} = 4$$

$$\text{या, } \frac{1}{\sin 2x} - \frac{\sqrt{3}}{\cos 2x} = 4$$

$$\text{या, } \frac{\cos 2x - \sqrt{3} \sin 2x}{\sin 2x \cdot \cos 2x} = 4$$

$$\text{या, } \cos 2x - \sqrt{3} \sin 2x = 4 \sin 2x \cdot \cos 2x$$

$$\text{या, } \cos 2x - \sqrt{3} \sin 2x = 2 \sin 4x$$

$$\text{या, } \frac{1}{2} \cos 2x - \frac{\sqrt{3}}{2} \sin 2x = \sin 4x \text{ [सिद्धावस्थाक 2 द्वारा अज कले]}$$

$$\text{या, } \sin \frac{\pi}{6} \cos 2x - \cos \frac{\pi}{6} \sin 2x = \sin 4x$$

$$\text{या, } \sin\left(\frac{\pi}{6} - 2x\right) = \sin 4x$$

$$\text{वा } \frac{\pi}{6} - 2x = m\pi + (-1)^m \cdot 4x$$

$$\text{वा, } (-1)^m \cdot 4x + 2x = \frac{\pi}{6} - m\pi$$

$$\text{वा, } x \{ (-1)^m \cdot 4 + (-2) \} = \frac{\pi}{6} - m\pi$$

$$\text{वा, } x = \frac{\frac{\pi}{6} - m\pi}{\{ (-1)^m \cdot 4 + 2 \}}$$

विचित्र m द्वि क्रान विषाद संख्या श्ले, $m=2n$ श्ले,

$$\therefore x = \frac{\frac{\pi}{6} - 2n\pi}{(-1)^{2n} \cdot 4 + 2}$$

$$= \frac{\frac{\pi}{6} - 2n\pi}{6}$$

$$= \frac{\pi}{36} - \frac{n\pi}{3},$$

विचित्र m द्वि क्रान विषाद संख्या श्ले, $m=2n+1$ श्ले,

$$\therefore x = \frac{\frac{\pi}{6} - (2n+1)\pi}{(-1)^{2n+1} \cdot 4 + 2}$$

$$= \frac{\frac{\pi}{6} - 2n\pi - \pi}{-1 \cdot 4 + 2}$$

$$= \frac{-\frac{5\pi}{6} - 2n\pi}{-4 + 2}$$

$$= \frac{-\frac{5\pi}{6} - 2n\pi}{-2}$$

$$= \frac{5\pi}{12} + n\pi$$

$$\therefore x = \frac{\pi}{36} - \frac{n\pi}{3}, \frac{5\pi}{12} + n\pi$$

सामान्य n द्वि क्रान शून्य वा सिकोला शून्य संख्या।

ଉତ୍ତର: ଅସ୍ମର ସିଦ୍ଧି

ଦେଖା ଗାଏ, $f(x) = \sin x$

$$\begin{aligned}\therefore f\left(\frac{\pi}{2} - \theta\right) &= \sin\left(\frac{\pi}{2} - \theta\right) \\ &= \cos\theta\end{aligned}$$

$$\begin{aligned}\text{ସି: } f\left(\frac{\pi}{2} - 2\theta\right) &= \sin\left(\frac{\pi}{2} - 2\theta\right) \\ &= \cos 2\theta\end{aligned}$$

$$\text{ଦିଅନ୍ତୁ, } 1 + f\left(\frac{\pi}{2} - \theta\right) + f\left(\frac{\pi}{2} - 2\theta\right) = 0$$

$$\text{ଦା, } 1 + \cos\theta + \cos 2\theta = 0$$

$$\text{ଦା, } 1 + \cos\theta + 2\cos^2\theta - 1 = 0$$

$$\text{ଦା, } \cos\theta (1 + 2\cos 2\theta) = 0$$

$$\text{କିନ୍ତୁ, } \cos\theta = 0$$

$$\therefore \theta = (2n+1)\frac{\pi}{2}$$

$$\text{କିନ୍ତୁ, } 1 + 2\cos\theta = 0$$

$$\text{ଦା } \cos\theta = -\frac{1}{2}$$

$$\text{ଦା, } \cos\theta = \cos\frac{2\pi}{3}$$

$$\therefore \theta = 2n\pi \pm \frac{2\pi}{3}$$

ଯେହ୍ନେ n କିମ୍ବା θ କିମ୍ବା ସମସ୍ତଙ୍କ ଦା ମାଧ୍ୟମରେ ସୂର୍ଯ୍ୟ ସଂଖ୍ୟା ।

ଦିଅନ୍ତୁ $n = 0, 1, 2, 3, \dots$ - - - କିମ୍ବା ସଂଖ୍ୟା,

$$n=0 \text{ ଥରେ, } \theta = \frac{\pi}{2}, \pm \frac{2\pi}{3}$$

$$n=1 \text{ ଥରେ, } \theta = \frac{3\pi}{2}, \frac{8\pi}{3}, \frac{4\pi}{3}$$

$$n=2 \text{ ଥରେ, } \theta = \frac{5\pi}{2}, \frac{10\pi}{3}, \frac{11\pi}{3}$$

$$\therefore 0 < \theta < 2\pi \text{ ସୀମା ସଂଖ୍ୟା } \theta \text{ କିମ୍ବା ସମସ୍ତଙ୍କ } \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}, \frac{4\pi}{3}$$